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Constrained MIMO radar waveform design by peak and total power

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Abstract

In this study, we discuss the optimal power allocation for designing MIMO radar waveforms, taking into account both peak and total power limitations. Both methods involve estimating the target impulse response, one by reducing the mean square error and the other by maximizing the mutual information between the target impulse response and the reflected waveforms. The radar sender is believed to be aware of the target's second-order statistics. Typically, the transmitter's total power restriction dictates how much of the available power is distributed to each of the antennas. However, the dynamic range and peak output of the power amplifier at each antenna is severely constrained by the huge power differences across the transmit antenna. In actuality, the maximum allowable peak power for any given antenna is the same across the board. Consequently, the peak power limitation of the transmit antennas should be taken into account. In order to limit the power amplifier's dynamic range at each transmit antenna, a new generalized constraint that satisfies both the peak power restriction and the average total power constraint has been developed. As a specific example of the sum power limitation, p = 1, the waterfalling idea of optimum power allocation holds. Numerical solutions are found using a nested Newton-type technique, whereas the ideal solution is determined using the Karush-Kuhn-Tucker (KKT) method, which maximizes mutual information while reducing mean square error. Simulation findings reveal that at low signal-to-noise ratios, the detection performance of the system improves when both the total and peak power limits are taken into account.

Introduction

MIMO radar, which uses multiple inputs and outputs, is a promising new development in the field of radar technology. More than 50 years ago, Woodward and Davies [1,2] advocated using information theory in radar. According to [3], it is best to maximize the mutual information (MI) between the extended target reflection and the received signal if the latter follows a Gaussian distribution. It's the first-time information theory has been used in the creation of a radar waveform, so far as we know. In [4], the parameters of many targets are estimated and tracked using a radar waveform designed using an information theoretic technique. For information-theory-based sensing applications like adaptive radar, the authors of [5] have presented a criterion for waveform selection. Information theoretic and estimation theoretic criteria for optimum waveform design have been used in recent research in the field of radar target detection and classification. Waveform design for multi-input multi-output (MIMO) radar (e.g., see [7–15]) has been the subject of previous studies (e.g., [6]), which sought to optimize for both maximum MI and minimal mean square error (MMSE). It was shown that the optimal solution obtained using these two distinct criteria is equivalent. Even an asymptotic formulation [6] based just on power spectral density (PSD) information confirms this. However, comprehensive understanding of the PSD may be challenging in actual use. The use of robust techniques, which account for modelling uncertainty in the design phase [16], may help in such a situation. Seem like a lot of fun. Information theoretic and estimate theoretic criteria are applied in the construction of the best signal for estimating correlated MIMO channels in [17].

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Notation

Matrices and vectors are denoted by the bold capital and lowercase characters, respectively. The transpose and complex conjugate transpose of a matrix are indicated by the superscripts'and, respectively. The determinant and trace of a matrix are shown by the notation det. and tr. The notation " " represents the Euclidean norm of a vector, whereas the notation diagram represents a diagonal matrix whose diagonal is the vector a. The notation N m; R stands for a complex Gaussian distribution with mean m and covariance matrix R. Last but not least, the positive value of an is represented by (a) +, where (a) + = max[0,a].

A Model of the System

Think of a MIMO radar that can cover a larger area by using N receiving antenna elements and M sending antenna elements. It is believed that the target is a location somewhere between the antennas being used for transmission and reception. The part of the received signal at the nth antenna element at time instant k is given by

$$y_n(k) = \sum_{i=1}^{M} h_{in} s_i(k) + \xi_n(k), \quad k = 1, \ldots, K,$$

where is(k) represents the transmit signal at the itch transmit antenna, him is the target impulse response from the itch transmit antenna to the nth receive antenna, and ξn(k) is the noise in the nth receive antenna. The components of the noise vector are assumed to be independent

and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance σξ 2 . In vector form, the signal model is written as

Waveform design

with p-norm power constraint According to Hadamard's inequality, the optimal solution of (11) and (14) can be achieved when (IM + ΛD)

$$(\mathbf{D} + \Lambda^{-1})^{-1}$$

Hadamard's inequalities for the determinant and trace of an n × n positive semidefinite Hermitian matrix A are

$$\det(\boldsymbol{A}) \le \prod_{i=1}^{n} a_{ii},$$

$$\operatorname{tr}(\boldsymbol{A}^{-1}) \ge \sum_{i=1}^{n} \frac{1}{a_{ii}},$$

where aii is the ith diagonal element of A, and equality is achieved in both cases if and only if A is diagonal [27]. Thus, D = XHX must be a diagonal matrix with nonnegative elements $d_{ii} \ge 0, \forall i \in [1, M]$.

. Now, the mutual information in (10) can be written as

$$I(\boldsymbol{Y}; \boldsymbol{H}/\boldsymbol{S}) = \log[\det(\boldsymbol{I}_M + \boldsymbol{\Lambda}\mathbf{D})].$$

It can be shown that (16) is concave as a function of D [28]. Similarly, the minimum mean square error in (13) can be written as

$$\text{MMSE} = \operatorname{tr}\left\{ (\boldsymbol{D} + \boldsymbol{\Lambda}^{-1})^{-1} \right\}.$$

The MMSE function in (17) is convex as a function of D [18]. If D = XHX should be a diagonal matrix, the columns of X should be orthogonal. Hence, X is factored as [29]

$$\boldsymbol{X} = \varphi \mathbf{D}^{1/2},$$

where the columns of φ are orthonormal. As X = SU, the transmitted signal matrix is given by

$$\boldsymbol{S} = \varphi \boldsymbol{D}^{(1/2)} \boldsymbol{U}^H,$$

$$\boldsymbol{S} = \varphi(\operatorname{diag}(d_{11}, d_{22}, \ldots, d_{MM}))^{(1/2)} \boldsymbol{U}^H,$$

here dii is the diagonal element of D. The two problems given in (11) and (14) are convex optimization problems that can be solved using the KKT optimality conditions [28].

Detection performance - Neyman-Pearson detector

The MIMO radar detection problem can be formulated as a binary hypothesis test as

$$\begin{aligned} \mathcal{H}_0 &: \boldsymbol{Y} = \xi, && \text{no target,} \\ \mathcal{H}_1 &: \boldsymbol{Y} = \boldsymbol{SH} + \xi, && \text{target exists.} \end{aligned}$$

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The probability density functions (pdfs) of Y under H0 and H1 are given by

$$p_o(\boldsymbol{Y}) = \frac{1}{\pi^{KN}\det^N\left(\sigma_\xi^2 \boldsymbol{I}_K\right)} \exp\left\{-\mathrm{tr}\left[\left(\sigma_\xi^{-2}\boldsymbol{I}_K\right)\boldsymbol{Y}\,\boldsymbol{Y}^H\right]\right\}$$

$$p_1(\boldsymbol{Y}) = \frac{1}{\pi^{KN}\det^N\left(\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right)} \times \exp\left\{-\mathrm{tr}\left[\left(\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right)^{-1}\boldsymbol{Y}\,\boldsymbol{Y}^H\right]\right\},$$

respectively. The log-likelihood becomes

$$l(Y) = \log\log\frac{p_1(Y)}{p_o(Y)} = \sum_{k=1}^{N}\boldsymbol{y}_k^*\left[\sigma_\xi^{-2}\boldsymbol{I}_K - \left(\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right)^{-1}\right]\boldsymbol{y}_k^T + c_l$$

Where

$$c_l = N\left[\log\det\left(\sigma_\xi^2\boldsymbol{I}_K\right) - \log\det\left(\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right)\right]$$

Is a constant term independent of Y. The optimal Neyman-Pearson detection statistics is given by

$$T(\boldsymbol{Y}) = \sum_{k=1}^{N}\boldsymbol{y}_k^*\left[\sigma_\xi^{-2}\boldsymbol{I}_K - \left(\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right)^{-1}\right]\boldsymbol{y}_k^T.$$

If T(Y) exceeds a given threshold, a target exists. To find the detection threshold, we have

$$\boldsymbol{y}_k^T \sim \begin{cases} \mathcal{CN}\left(0, \sigma_\xi^2\boldsymbol{I}_K\right), & \mathcal{H}_0 \\ \mathcal{CN}\left(0, \boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right), & \mathcal{H}_1 \end{cases}.$$

Let

$$\mathbf{P} = \sigma_\xi^{-2}\boldsymbol{I}_K - \left(\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi^2\boldsymbol{I}_k\right)^{-1}.$$

$$2\boldsymbol{y}_k^*\mathbf{P}\boldsymbol{y}_k^T \sim \sum_{j=1}^{K}\alpha_j^{(i)}\chi_2^2(j).$$

Under $\mathcal{H}_i, i = 0, 1$, where $\alpha_j^{(i)}$ is the jth eigenvalue $\mathbf{P}^{1/2}(\gamma\boldsymbol{S}\boldsymbol{R}_H\boldsymbol{S}^H + \sigma_\xi{}^2\boldsymbol{I}_k)\mathbf{P}^{1/2}$, $\gamma = 0$ under $\mathcal{H}_0$ and $\gamma =$ under $\mathcal{H}_1$. Therefore we have

$$2\,T(\boldsymbol{Y}) = 2\sum_{k=1}^{N}\boldsymbol{y}_k^*\mathbf{P}\boldsymbol{y}_k^T \sim \sum_{k=1}^{K}\alpha_k^{(i)}\chi_{2N}^2(k)$$

under Hi; i ¼ 0; 1 . The test statistics is the weighted sum of chi-squares. It is approximated as gamma distribution [32]. If Cq are real positive constants and Nq are independent standard normal random variables, ∀q = 1,…, K, then the pdf of the gamma approximation of

$$R = \sum_{q=1}^{K}C_qN_q{}^2 \text{ is given as}$$

$$f_R(r, a, b) = \frac{r^{a-1}e^{-\frac{r}{b}}}{b^a\Gamma(a)},$$

where the parameters a and b are given as

$$a = \frac{1}{2}\left[\frac{\left(\sum_{q=1}^{K}C_q\right)^2}{\sum_{q=1}^{K}C_q^2}\right],$$

$$b = \left[\frac{1}{2}\left(\frac{\sum_{q=1}^{K}C_q}{\sum_{q=1}^{K}C_q^2}\right)\right]^{-1},$$

where Γ is the gamma function defined as

For the test statistics in (43), Cq corresponds to αk (i) and Nq 2 corresponds to χ2 2N ð Þ k χ. After approximating the pdf using the gamma density, the probability of detection (PD) and the probability of false alarm (PFA) are defined as

$$P_{\mathrm{D}} = \int_{\gamma}^{\infty} t^{a_{H1}-1}\frac{e^{-\frac{t}{b_{H1}}}}{b_{H1}^{a_{H1}}\Gamma(a_{H1})}dt,$$

$$P_{\mathrm{FA}} = \int_{\gamma}^{\infty} t^{a_{H0}-1}\frac{e^{-\frac{t}{b_{H0}}}}{b_{H0}^{a_{H0}}\Gamma(a_{H0})}dt,$$

where aH0 and bH0 are the parameters of the gamma density for null hypothesis ð Þ H0 and aH1 and bH1 are the parameters of the gamma density for alternate hypothesis ð Þ H1 . It is known that

$$P_{\mathrm{FA}} = Pr(T(\boldsymbol{Y}) > \gamma \mid \mathcal{H}_0).$$

For a given value of PFA, the threshold γ is calculated using (49), and the probability of detection is calculated using (48) with the functions available in MATLAB.

numerical example

This section provides numerical examples to illustrate the performance of MIMO radar waveform with combined peak and sum power constraints. A MIMO radar system with M = 5 transmit and N = 5 receive antenna system is considered. First, we consider the power allocation among the transmit antennas. Figure 1 illustrates

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the optimal transmitting power on one of the antennas for 100 different target impulse response realizations for various values of the norm, p: 1. p = 1, SPC: This case corresponds to the waterfilling strategy, and power is allotted in proportion to the quality of the target mode. More power is allotted to a better mode. For low values of total power, no power is allotted to poor quality modes. As shown in Figure 1, the transmit power fluctuates as much as 4 W in the transmit antenna under the sum power constraint. 2. 1 < p < ∞, peak and sum power constraints (PSPC): When the value of p is appropriately chosen, this satisfies the sum power constraint of the whole system and the peak power constraint of the individual antenna. If the individual power

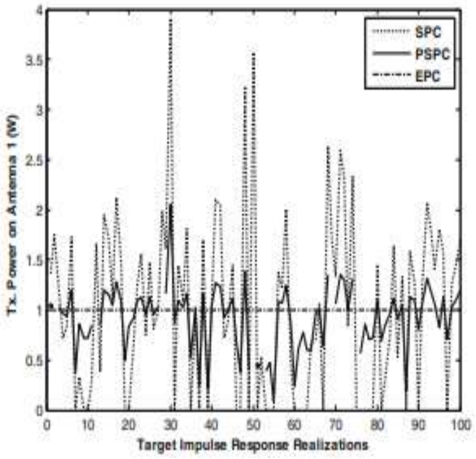

Figure 1 Transmit power across the first antenna.

If the total power limit is set to 2 W, then the value of p is 2.32 according to (9), satisfying both the sum power limit and the individual power limit. Figure 1 shows that the limitation imposed on the power of each individual antenna necessitates that the same 5 W be spread throughout all five antennas. In addition, each antenna is checked to make sure its peak power output doesn't go over 2 W. After four to six iterations, the numerical procedure converges to within eight significant figures, provided that the beginning value of the outer iteration 0 and the initial values for calculating qi 1 ((0)) are suitably selected. Third, in an EPC where p =, we have: In this scenario, all antennas used for transmission get the same percentage of the total power.

Conclusions

We have looked at how well MIMO radar waveforms work when peak and total powers are limited. The second-order statistics of the extended target impulse response, which provides crucial information about the target's properties, is used for the creation of the optimal radar waveform for MIMO radar. Finding the best answers is the goal of the KKT method. When p is equal to one, the famous waterfalling concept is shown to be a limiting case. In the range 1p, it is proven that the power distribution may be computed numerically. Using a cumulative power limitation and a peak power restriction, we analyse the MI performance, MMSE performance, and detection performance. At low signal-to-noise ratios (SNRs), it has been shown that performance with combined peak and sum power limitations is better. Although not ideal, this restriction has real-world relevance. Therefore, it is proposed that a mixed method of power distribution be used. The sum power constraint is utilized when the signal-to-noise ratio (SNR) is low and the combined peak and sum power restriction is used when the SNR is high.

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